

Self-accelerating optical beams in highly nonlocal nonlinear media

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Abstract: We find self-accelerating beams in highly nonlocal nonlinear optical media, and show that their propagation dynamics is strongly affected by boundary conditions. Specifically for the thermal optical nonlinearity, the boundary conditions have a strong impact on the beam trajectory: they can increase the acceleration during propagation, or even cause beam bending in a direction opposite to the initial trajectory. Under strong self-focusing, the accelerating beam decomposes into a localized self-trapped beam propagating on an oscillatory trajectory and a second beam which accelerates in a different direction. We augment this study by investigating the effects caused by a finite aperture and by a nonlinear range of a finite extent.

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1. Introduction

The past few years have witnessed considerable research on self-accelerating beams and optical pulses. This concept was pioneered by Berry and Balazs in 1979 [1], who found a special solution for the Schrödinger equation, where a wave-packet describing a free particle is self-accelerating. In 2007, this idea was introduced into optics, and used to demonstrate experimentally self-bending beams which have the functional shape of an Airy function [2,3]. Since then, this phenomenon has drawn much interest, with applications ranging from trapping of particles along curved paths [4] and self-bending plasma channels [5] to ultrafast self-accelerating pulses [6] and Airy light bullets accelerating in both transverse dimensions and in time [7]. Until recently, nonlinearities have been perceived as an obstacle for supporting self-accelerating wavepackets: attempts to launch an Airy beam into Kerr media revealed that the beam shape varies - no longer propagating as accelerating shape-invariant entity [7–9], due to "nonlinearly induced distortions" [7]. However, recently our group has shown [10] that it is possible to generate self-accelerating nonlinearly-self-trapped beams in various types of nonlinear media, ranging from Kerr (where this idea was pioneered in 1989 [11]) and saturable Kerr-like media to quadratic nonlinearities. Related studies followed soon thereafter [12,13]. In principle, the formulation in [10] applies to all nonlinear media with a local response, hence it is likely that self-accelerating beams will be found in any local optical nonlinearities including in systems where two polarizations are coupled (e.g., the Manakov system). However, many soliton-supporting nonlinear systems display a highly nonlocal response, for example thermal glasses [14,15], liquid crystals [16,17], charge carriers in semiconductor amplifiers [18] and more. This raises the intriguing question: can self-accelerating solitons exist in nonlocal nonlinear media? This question becomes even more interesting in long-range nonlocal nonlinear media, where the response is strongly affected by boundary conditions, such that even highly localized soliton beams can be controlled from afar [15,19,20]. This issue is especially interesting for accelerating beams, because all accelerating wavepackets found thus far, in linear and nonlinear media, albeit being localized functions, they all have an infinite oscillating tail. Consequently, we expect accelerating beams in nonlocal nonlinear media to be strongly affected by boundary conditions, even from very far away.

In this paper, we do just that: we present self-accelerating beams in nonlocal nonlinear optical media, focusing specifically on the highly nonlocal thermal optical nonlinearity. We show that the non-accelerating thermal boundary conditions have a strong impact on the beam trajectory: they can increase the acceleration during propagation, or even cause beam bending in a direction opposite to the initial trajectory. Under strong self-focusing, the accelerating beam decomposes into a localized self-trapped beam, and a second beam which accelerates in a different direction.

2. Accelerating self-trapped (propagation-invariant) solutions

The thermal nonlinear optical effect is caused due to absorption of light by the medium, which raises the local temperature. In such a medium, the temperature modifies the refractive index, according to $\Delta n = \beta \Delta T$, $\beta = dn/dT$ [15], where β describes the temperature dependence of the refractive index, ΔT is the change in the temperature and Δn is the change in the refractive index. The temperature obeys the heat equation, which under steady-state conditions, yields an equation for the change in the refractive index [15]:

$$\frac{\kappa}{\beta} \nabla^2 \Delta n = -\alpha |\psi|^2 \quad (1)$$

κ is the thermal conductivity, α is the linear absorption coefficient of the material, and ψ is the slowly-varying amplitude of the electromagnetic field. The sign of β determines if the medium is of the self-focusing or self-defocusing type. In many relevant materials (e.g., lead glass), α is very small, such that absorption can be safely neglected for fairly large propagation distances, while the thermal nonlinearity is still very strong [15]. For simplicity, we henceforth analyze one-dimensional beams propagating in this nonlinear medium. The evolution of the slowly-varying envelope $\psi(x, z)$ of such beams is described by the nonlinear paraxial equation:

$$i\psi_z + \frac{1}{2k} \psi_{xx} + \frac{k\Delta n}{n_0} \psi = 0 \quad (2)$$

where z is the optical axis, x is the transverse direction, $k = 2\pi n_0 / \lambda$ is the wavenumber, λ is the vacuum wavelength, n_0 is the background refractive index of the medium, and Δn satisfies Eq. (1). As shown in [19], when the refractive index change Δn is not too large (~ 0.01 and smaller), the evolution is slow and adiabatic, such that the z -derivatives in Eq. (1) can be safely neglected. To find self-accelerating beams in such a medium, we use the same method used in [10] to find accelerating beams in media with local nonlinear response. We seek self-trapped self-accelerating solutions that satisfy $|\psi(x, 0)| = |\psi(x - f(z), z)|$, meaning that the beam is propagating along the curve $x = f(z)$, while maintaining its exact intensity structure. Such a solution is a self-trapped solution in the accelerating frame of reference. We change variables $\hat{x} = x - f(z)$, transforming Eq. (2) to the accelerating frame, which yields

$$i\hat{\psi}_z - if'(z)\hat{\psi}_{\hat{x}} + \frac{1}{2k} \hat{\psi}_{\hat{x}\hat{x}} + \frac{k\Delta\hat{n}}{n_0} \hat{\psi} = 0 \quad (3)$$

We seek propagation-invariant solutions for Eq. (3), namely, $\hat{\psi}(\hat{x}, z) = u(\hat{x}) e^{i\Gamma(\hat{x}, z)}$, where u is a real function. After setting to zero the coefficients of terms that diverge if $u(\hat{x})$ goes through zero, we obtain

$$\frac{1}{2k} u'' + \frac{k\Delta\hat{n}}{n_0} u - f'' k (\hat{x} - x_1) u = 0 \quad (4)$$

with the condition $f'' k = \text{const}$, which restricts $f(z)$ to be parabolic. Other trajectories could be possible, if the additional terms of the form $1/u^2$, $1/u^4$ are kept. However, to be physical, such solutions should not attain zero value anywhere. Keeping in mind that all accelerating beams in lossless media found thus far possess oscillations that go through zero [1,21], it seems unlikely that additional, non-parabolic, trajectories could exist for self-

accelerating beams displaying propagation-invariant evolution. Hence, we focus here on beams propagating along parabolic trajectories only. We apply transformation of variables to

Eqs. (1) and (4), finding $\tilde{x} = A^{1/3}(\hat{x} - x_1)$, $\tilde{u} = \frac{\sqrt{2k^2}}{A^{2/3}}u$, $\Delta\tilde{n} = \frac{2k^2}{A^{2/3}n_0}\Delta n$, where $A = 2k^2 f''$

and $c = \frac{\beta\alpha}{\kappa n_0}$, which yields

$$\Delta\tilde{n}'' = -c|\tilde{u}|^2 \quad (5)$$

$$\tilde{u}'' + \Delta\tilde{n}\tilde{u} - \tilde{x}\tilde{u} = 0 \quad (6)$$

The Green function of Eq. (5), for trivial boundary conditions, is $G(\tilde{x}, \tilde{x}') = -const \cdot |\tilde{x} - \tilde{x}'|$. Using this we rewrite Eq. (6):

$$\tilde{u}'' - c \left[\int_{-L}^L |\tilde{x} - \tilde{x}'| |u(\tilde{x}')|^2 d\tilde{x}' \right] \tilde{u} - \tilde{x}\tilde{u} = 0 \quad (7)$$

To approximate the integral, we assume that the self-trapped solution has at least one decaying tail. Therefore, for some x_0 , (with $x_0 > 0$, without loss of generality) every $\tilde{x} > x_0$ satisfies $|u(\tilde{x})|^2 \rightarrow 0$. We approximate the integral term for $\tilde{x} > x_0$:

$$-\int_{-L}^L |\tilde{x} - x'| |u(x')|^2 d\tilde{x}' \rightarrow -\int_{-L}^L (\tilde{x} - x') |u(x')|^2 dx' = -\tilde{x}P + X_{CM} \quad (8)$$

where $P = \int_{-L}^L |u(x')|^2 dx'$ and $X_{CM} = \int_{-L}^L x' |u(x')|^2 dx'$. This approximation is valid for any solution having at least one decaying tail. Under the condition $cP \ll 1$, which physically implies a weak nonlinearity, Eq. (7) is similar to the Airy equation $\tilde{u}'' - \tilde{x}\tilde{u} = 0$, whose solutions are the Airy functions. This suggests that, for every such small cP , the refractive index change $\Delta\tilde{n}(x)$ is approximately linear in \tilde{x} for any $\tilde{x} > x_0$. Hence, we expect the solution on the side of the decaying tail of \tilde{u} to be similar to the Airy function, which is the eigen-function of a potential structured as a linear gradient.

We solve Eq. (5) and Eq. (6) numerically in the interval $[-a, a]$, and find the eigen-functions (the propagation-invariant solutions in the accelerated frame). As sketched in Fig. 1, the boundary conditions for $\Delta\tilde{n}$ are $\Delta\tilde{n}(\tilde{x} = -a) = \Delta\tilde{n}(\tilde{x} = a) = 0$ (arising from $T(\tilde{x} = \pm a) = T_0$, T_0 being the temperature everywhere in the absence of light). The boundary conditions for the field amplitude, $\tilde{u}(\tilde{x} = -a)$ and $\tilde{u}(\tilde{x} = a)$, can be any non-zero finite value. We choose this value to be such that does not make \tilde{u} diverge. A typical solution is shown in Fig. 1.

Notice that the change in the refractive index, $\Delta\tilde{n}$, is approximately linear near the main lobe, as we expected. This implies that, near the main lobe, the solution is similar to the Airy function. As $\tilde{x} \rightarrow -a$, the self-trapped solution differs from the Airy function, mainly due to the boundary conditions.

3. Propagation dynamics of accelerating self-trapped beams

Having found the self-accelerating self-trapped solutions, we study their propagation dynamics. It is important to emphasize that these propagation-invariant beams were found under boundary conditions $\Delta\tilde{n}(\tilde{x} = -a) = \Delta\tilde{n}(\tilde{x} = a) = 0$, which are z -independent in the accelerating frame. However, physically, this will almost never be the case in experiments: to do that one would have to pre-select a given trajectory, and subsequently cut the nonlinear

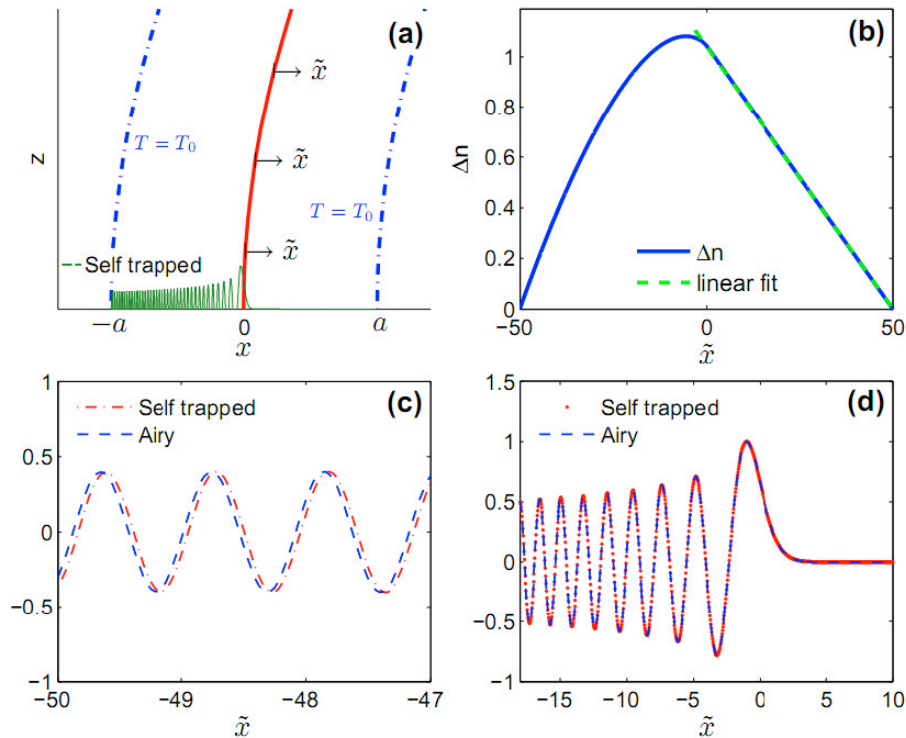


Fig. 1. (a) Sketch of the physical system and the boundary conditions used to find the self-accelerating self-trapped solutions, with a typical beam shown at $z = 0$. To find these solutions, which are propagation-invariant in the accelerating frame, we assume that the boundaries of the nonlinear medium accelerate with the beam, such that the temperature at these boundaries is constant. (b-d) An example of a self-accelerating self-trapped solution in the thermal optical nonlinearity of the self-focusing type: (b) normalized refractive index change $\Delta\tilde{n}(\tilde{x})$ supporting the self-accelerating self-trapped beam, (c,d) normalized amplitude of such beam (red dotted) compared to the Airy function (blue dashed), far from main lobe (c) and at the vicinity of the main lobe (d).

sample specifically for that trajectory. Rather, experimentally one would typically use a rectangular sample with z -independent boundary conditions in the non-accelerating lab frame. In what follows, we study the impact of such non-accelerating boundary conditions on the accelerating beam that was chosen such that it is self-trapped (propagation-invariant) under accelerating boundary conditions. This setting conforms to typical experiments with self-trapped beams in nonlocal nonlinear media [15,19,20,22]. From previous work with boundary force effects on solitons in such media [19], we expect the propagation dynamics of the accelerating beams to be strongly affected by boundary conditions, even from far away from the beam. As we show below, by controlling the power of the beam we change the strength of the nonlinearity, and through that the impact of the boundary conditions on the propagation dynamics. The simulations are carried out using the techniques described in [15,19,20,22]. In all cases, the initial condition (the launch beam) is the self-consistent solution of the type shown in Fig. 1 (truncated only by the boundaries of the sample), which would exhibit self-accelerating self-trapped propagation in the same system, under accelerating boundary conditions. Typical simulation results are depicted in Fig. 2. The physical parameters used for the examples shown in Fig. 2 and Fig. 3 are the optical wavelength $\lambda = 0.75\mu\text{m}$ in vacuum, the background refractive index $n_0 = 1.5$, $|c| = 2 \cdot 10^{-5}$, where the sign of c determines whether the nonlinearity is of the self-focusing or the self-defocusing type.

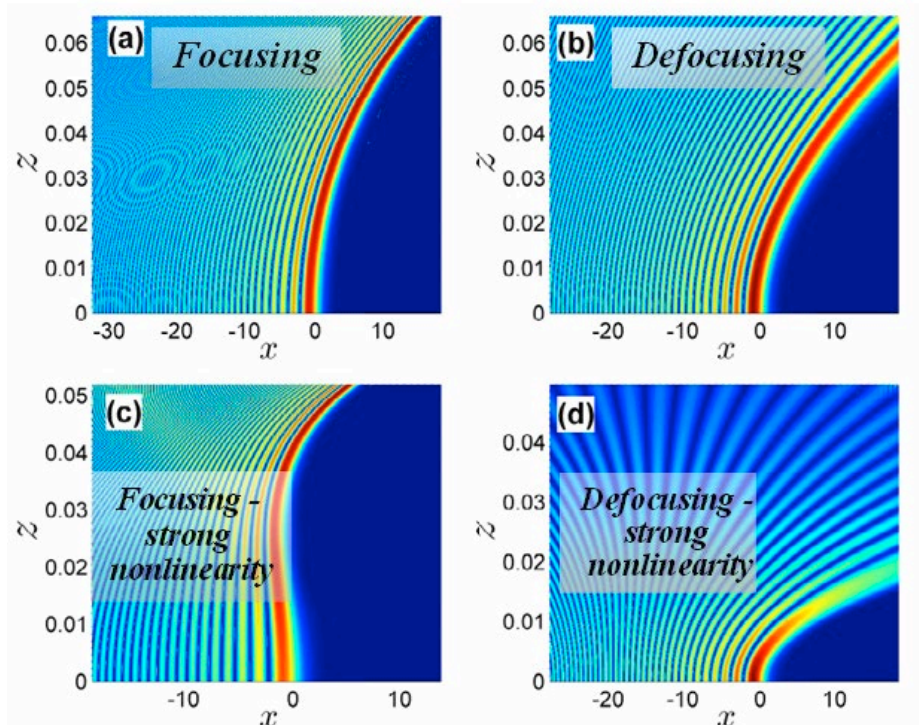


Fig. 2. Propagation dynamics occurring when the beam from Fig. 1, with the width of the main lobe being $50\ \mu\text{m}$, is launched to propagate in a rectangular sample of $7.5\ \text{mm}$ width. The boundary conditions are z -independent in the lab frame. Such beam would exhibit self-accelerating self-trapped propagation under accelerating boundary conditions. Four generic cases are depicted. (a) Focusing nonlinearity. The power is such that the maximum change in the refractive index is $4.2 \cdot 10^{-4}$. After propagation distance of $6.6\ \text{cm}$ the main lobe is shifted by $400\ \mu\text{m}$ in the x direction. (b) Defocusing nonlinearity. The power is such that the maximum (negative) change in the refractive index is as small as $-6 \cdot 10^{-7}$. After propagation distance of $6.6\ \text{cm}$ the main lobe is shifted by $600\ \mu\text{m}$ in the x direction. (c) Strong focusing nonlinearity, and (d) strong defocusing nonlinearity, under the input conditions as in (a) and (b), with the power adjusted such that the maximum index changes are $2.9 \cdot 10^{-3}$ and $-1.15 \cdot 10^{-5}$, respectively. In all plots x is normalized as described in the text, and the z units are meters.

We tested the stability of the self-accelerating solution numerically under random noise in a magnitude of 10% of the amplitude of the main lobe. We find that the accelerating beams are virtually unaffected by the noise, for both the focusing and defocusing cases. This is in fact excepted from highly nonlocal nonlinearities, which always tend to suppress localized perturbations in the beam and/or the index change.

The results depicted in Fig. 2 show that varying the power of the beam changes the beam trajectory. Notice that the beam is no longer propagation-invariant in the accelerating frame of reference. The reason for this is the thermal boundary conditions. Namely, the boundary conditions are symmetric at $z = 0$, however, as the beam is propagating, it undergoes spatial acceleration (beam bending), which inherently makes the boundaries to become non-symmetric with respect to the beam. As we know from [19], asymmetric boundary conditions exert forces on the beam, which affect the beam trajectory. The complex dynamics shown in Fig. 2 arises from this interplay between interference effects predesigned to support a particular accelerating trajectory and boundary forces exerted on the beam, in highly nonlocal nonlinear media. Clearly, these forces have a major impact on the beam trajectory, leading to

accelerating trajectories that are very different from the parabolic trajectories found for the Airy beams propagating in linear media. For the self-focusing case, the acceleration decreases as the beam power is increased, and for strong self-focusing the trajectory becomes non-parabolic. This is a clear manifestation of the underlying nonlocal nonlinearity, because in nonlinearities where Δn has a local response, all trajectories are parabolic. Furthermore, all accelerating beams supported by linear interference effects must have convex trajectories [21], in a sharp contrast to the “wiggly beam” of Fig. 2c, which exhibits a non-convex trajectory. Also, notice that under strong self-focusing the lobes of the accelerating beam become narrower as the beam is propagating. For the self-defocusing case (Fig. 2d), the acceleration increases as the beam power is increased, and for a strong defocusing nonlinearity all the lobes of the beam expand. Thus, controlling the power of the optical beam allows us to change the trajectory (spatial acceleration) of the beam, by virtue of boundary force effects arising from the long-range nonlocal nature of the thermal nonlinearity.

Another very interesting phenomenon occurs in the self-focusing case. For very strong nonlinearity, the original beam decomposes into two beams, as depicted in Fig. 3c: The original Airy-like beam that continues propagating on a different trajectory, until it loses all

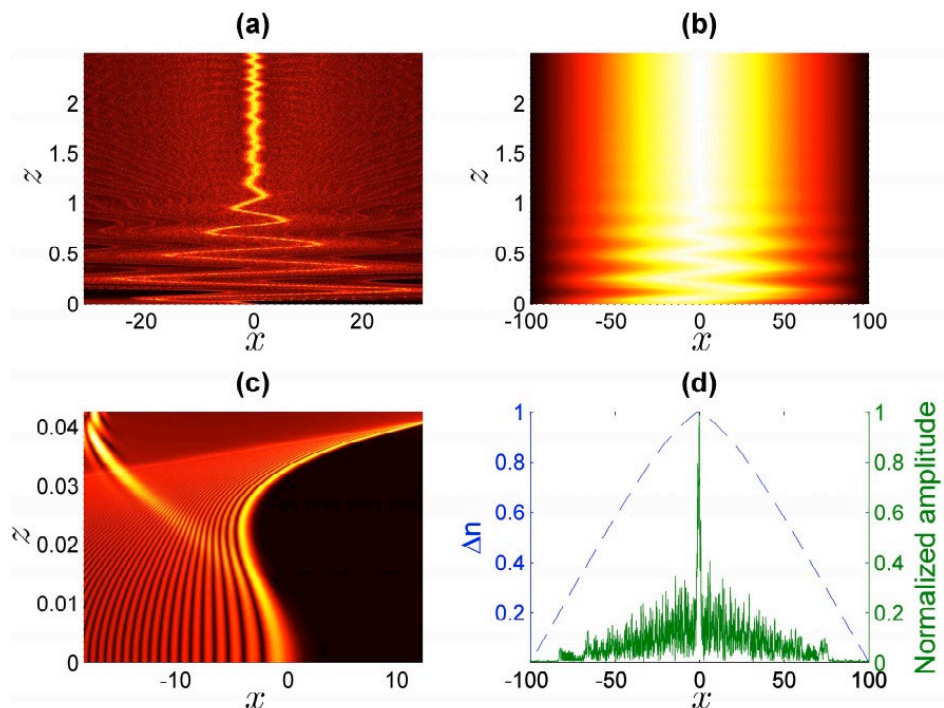


Fig. 3. Propagation dynamics of the launch beam from Fig. 1 under strong self-focusing nonlinearity. (a) Long range propagation dynamics showing how the Airy-like launch beam exhibits strong oscillations in its trajectory, and then transforming into a narrow self-trapped (soliton-like) beam propagating on approximately straight line. (b) Evolution of the nonlinear index change Δn , showing that it oscillates together with the beam, and then stabilizes to support a soliton. (c) Zooming into the first stage of evolution in (a) reveals that the beam decomposes into two beams, one transforming into a localized self-trapped beam propagating at an approximately straight line, while the other shedding power and disappearing. (d) Normalized profile of the highly localized self-trapped beam and of Δn (dashed line) supporting it at the plane $z = 1.82 \text{ m}$. The width of the main lobe of the input beam is $50 \mu\text{m}$, and the power is such that the maximum index change is $5.4 \cdot 10^{-3}$. The boundaries are located symmetrically 2.5 mm away from $x = 0$. In all plots x is normalized as described in the text, and the z units are meters.

its power as a result of the finite aperture of the beam, and a second, new, localized beam that has taken its power from the original beam. Notice that this second beam lacks any oscillating tails characteristic of the Airy-like wavepackets, which accelerate due to their structure. Indeed, this second beam is self-trapped by virtue of the self-focusing nonlinearity: it is a spatial soliton [23]. For strong self-focusing, the nonlinear term in Eq. (6) becomes dominant, causing focusing of the beam that overcomes the tendency of the beam to accelerate. This new self-trapped beam first moves on an oscillatory trajectory while radiating power during propagation. Later on, after that beam has gained enough power, the trajectory oscillations decrease, and the beam stabilizes around a constant power, becoming a spatial soliton propagating at an approximately straight line trajectory in the z direction. Such oscillatory propagation dynamics of solitons in this highly nonlocal nonlinear medium is familiar from earlier work [19]: it appears when the soliton is launched off-center, such that the boundaries exert asymmetric forces on the beam. What we have shown here is a fascinating nonlinear phenomenon in which a localized soliton is created from a wide beam with very long tails, that are affected by far away boundary conditions through a highly nonlocal nonlinearity.

4. Accelerating self-trapped beams carrying finite power

The self-accelerating beams discussed thus far in this article carry infinite power (truncated only by the boundaries of the sample). Physically, however, all beams must carry finite power. Accordingly, accelerating beams carrying finite power were analyzed theoretically in both the linear domain [2,3] and in local nonlinearities [10]. In this section, we study the propagation dynamic of accelerating beams carrying finite power in highly nonlocal nonlinear media. Specifically, we are interested to elucidate the influence of the width of the aperture (which is what makes the beam finite) on the propagation dynamic of the beam. We study numerically the propagation dynamic of such finite-power beams under self-focusing and self-defocusing, for varying power levels. It is important however to distinguish between the linear diffractive evolution associated with launching finite beams (namely, that the acceleration occurs only for a finite distance determined by the aperture [2,3,10]), and the effects caused by the nonlocal response of the nonlinearity. Consequently, we simulate finite beams whose initial width (aperture) is considerably narrower than the width of the nonlinear sample (2a in Fig. 1).

We find that, under self-focusing conditions, such self-accelerating finite beam exhibits all the phenomena described above with infinite beams, including the observation that, as the power of the beam is increased the acceleration decreases, and the intriguing effects occurring under strong self-focusing (Fig. 3). Likewise, under self-defocusing, as the beam power is increased the acceleration increases, similar to what happens with infinite beams.

The effects associated with the highly nonlocal nature of the nonlinearity do give rise to one important effect caused by the aperture: changing the aperture changes the distance of the beam from the boundaries. Hence, when the aperture is wider the boundary forces are more dominant and have a more significant influence on the acceleration of the beam. A typical example of the propagation dynamic of such a finite self-accelerating beam is displayed in Fig. 4: a beam of wavelength $\lambda = 0.75\mu\text{m}$, going through an aperture of 1.8mm , which allows 55 lobes with width varying from $50\mu\text{m}$ (main lobe) to $10\mu\text{m}$ (narrowest lobe). All the physical parameters of the beam and of the sample are exactly as for the beam in Fig. 2a. The beam enters a sample of 7.5mm width, the background refractive index is $n_0 = 1.5$, and $|c| = 2 \cdot 10^{-5}$. The power is the same as of the beam in Fig. 2a. However, for this finite beam the same power gives rise to a change in the refractive index which at its peak is as large as $5.4 \cdot 10^{-3}$, one order of magnitude higher than that of the infinite beam. The reasoning is simple: the same power is now distributed over the finite aperture, which is narrower than the sample width, hence the higher nonlinear effects. Consequently, the main nonlinear effect caused by the finite width of an accelerating launch beam is that all the nonlinear effects are

“scaled up”: they occur at power levels lower than those required to exhibit the same effects with infinite beams.

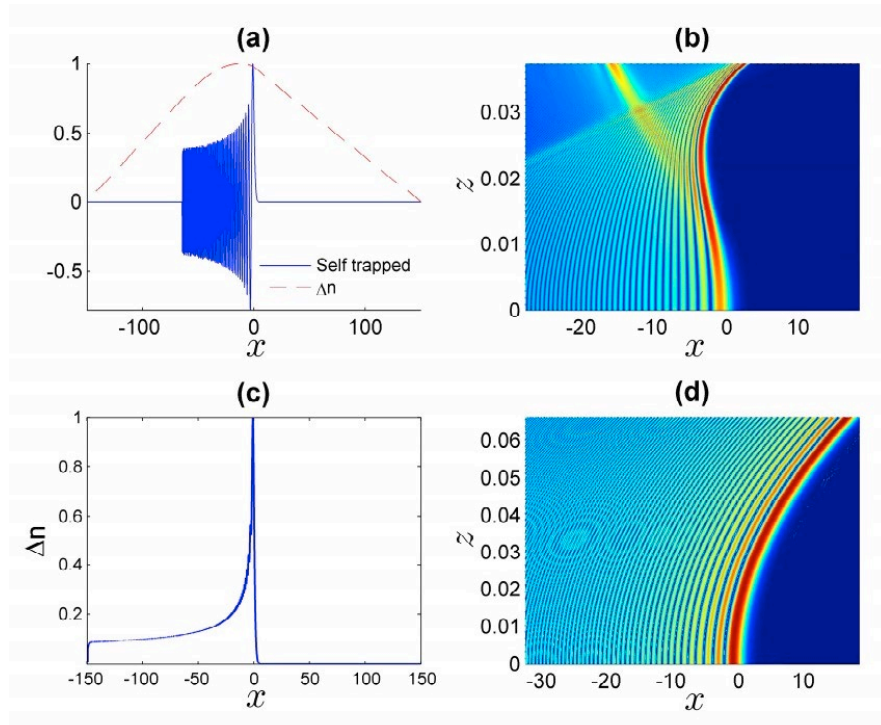


Fig. 4. (a) Normalized amplitude of a finite accelerating beam with an aperture of 1.8mm allowing 55 lobes (solid line), and the normalized change in the refractive index induced by such beam at $z = 0$ (dashed line). (b) Propagation dynamics occurring when the finite Airy-like beam of (a) is launched into a rectangular sample of 7.5mm , under self-focusing nonlinearity. The power is the same as the beam in Fig. 2a, however the maximum change in the refractive index is now much higher ($5.4 \cdot 10^{-3}$), and the finite launch beam decomposes into two separate beams, similar to the behavior of an infinite beam under strong self-focusing. (c) Normalized change in refractive index at $z = 0$ under nonlinearity with a range of nonlocality of $20\ \mu\text{m}$. (d) Propagation dynamic occurring when this beam with a main lobe width of $50\ \mu\text{m}$ is launched into a rectangular sample of 7.5mm width, under nonlinearity with a finite range of nonlocality of $20\ \mu\text{m}$. The power of the beam is the same as the beam in Fig. 2a. After propagation distance of 6.6cm the main lobe is shifted by $425\ \mu\text{m}$ in the x direction, which is more than in the infinite-range nonlocal case. In all plots x is normalized as described in the text, and the z units are meters.

5. Accelerating self-trapped beams in nonlinear media with a finite range of nonlocality

In the previous sections we dealt with a highly nonlocal nonlinearity, in which the nonlocal response has an infinite range. Here we consider accelerating beams propagating in a medium with nonlinearity having a finite range of nonlocality. Specifically, we are interested in the influence of the range of the nonlocality on the propagation of the beam. Such finite-range nonlocal nonlinearities occur in a variety of physical systems, such as liquid crystals [16,17], semiconductor amplifiers [18] and photorefractives [24–26]. In many cases, the equation describing the change in the refractive index under nonlocal nonlinearity with a finite range can be written as [27]:

$$-\frac{1}{l^2} \frac{\Delta n}{n_0} + \frac{\Delta n''}{n_0} = -c|u|^2 \quad (9)$$

Where c is defined above, and l describes the nonlocality range. We use Eq. (9) that describes a nonlocal nonlinearity with an exponential response function, instead of Eq. (5). When coming to explore the effects of the finite range of nonlocal response, the interesting domain is when l is comparable to the width of the lobes of the beam. If the range of the nonlocality is much greater than the width of the main lobe, the nonlinearity is highly nonlocal, and the propagation dynamic of the beam is similar to that displayed in the previous sections. If the range of the nonlocality is smaller than the width of the narrowest lobes, then the nonlinearity is effectively local and the propagation dynamic of the beam is known from previous work [10]. We therefore study the propagation of beams under nonlinearity with a varying range of nonlocality, in the interesting regime where the nonlocality range is comparable to the width of the beam lobes. Under self-focusing, we find that as the range of the nonlocality is larger the acceleration of the beam is smaller. This can be understood by the fact that the larger range of the nonlocality allows the boundaries to exert stronger forces on the beam, resisting the original tendency of the beam to accelerate. Additionally, the relaxation term in Eq. (9) reduces the maximum nonlinear index change, which also reduces the forces exerted by the sample boundaries. Figures 2c and 2d demonstrate the propagation dynamics of a beam entering a sample with a nonlinear response with a finite range of nonlocality. The parameters of the beam (power, wavelength) are equal to those of Fig. 2a. The range of the nonlocality is $20 \mu m$, which is smaller than the width of the main lobe yet greater than the width of the lobes far from the main lobe. The propagation dynamics is similar to that in an infinite-range nonlocal nonlinearity, except that the acceleration here is stronger due to lack of the forces exerted by the boundaries.

6. Conclusion

We have found self-accelerating self-trapped beams supported by the highly nonlocal thermal nonlinearity. We demonstrated the complex dynamic of an accelerating beam propagating under non-accelerating boundary conditions that exert forces on the beam. We showed that changing the nonlinearity strength allows control over the trajectories of the beam, to the extent that it gives rise even to trajectories that are completely different from those of the linear self-accelerating Airy beam. Moreover, we found a new surprising phenomenon created by the presence of high focusing nonlinearity: a very broad (“infinite”) launch beam decomposes into two beams, one of which becoming a localized spatial soliton, whose trajectory oscillates around a straight line, while the other beam is shedding power and eventually dies out. This picturesque phenomenon brings about a variety of intriguing questions: can the interaction between nonlinear accelerating beams produce spatial solitons? What would be their trajectories? Can they be pre-designed? We leave these ideas for future research.

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